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International Journal of Solids and Structures 41 (2004) 6937–6948

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

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Energetic conditions for interfacial failure in the vicinity of a matrix crack in brittle matrix composites

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Received 5 December 2003; received in revised form 22 May 2004
Available online 3 July 2004

Abstract

The nucleation of an interfacial crack is analysed in the vicinity of a matrix crack. The selected geometry is an axisymmetric fibre/matrix cell submitted to a tensile loading. For a given value of the matrix ligament, an energetic approach provides a nucleation condition comparing the ratio of the interfacial toughness over the matrix toughness to a critical value depending on the elastic mismatch between the fibre and the matrix and the decohesion length. An additional condition is needed to determine the decohesion length and an energetic condition and a strength condition are compared. Predictions of decohesion conditions are then presented in the case of a stationary or propagating matrix crack.

Keywords: Interfacial debonding; Crack deflection; Energetic analysis; Ceramic matrix composite

1. Introduction

Brittle ceramics can be made highly damage tolerant by combining them in fibre/matrix composite form. Deflection of matrix cracks to the fibre/matrix interface is essential for achieving a tough behaviour (Evans et al., 1991; Naslain, 1993). As a matter of fact, debonding relieves the stress on the fibre along the crack plane and allows crack bridging as the fibre is left intact behind the crack tip. In such composites, crack deflection is effected by a weak and compliant coating applied to the fibre before processing or formed in situ by fibre decomposition during matrix processing. The coating is thus a component of the composite system that must be engineered to promote a specific failure mechanism (Kerans et al., 2002). Enhanced understanding of crack deflection is necessary to allow this coating design. However, detailed fracture observations are difficult and the sequence of events for crack deflection in brittle matrix composites remains speculative.

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0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2004.05.044

Previous studies assume that debonding is delayed until the crack impinges on the interface. The crack can either propagate into the fibre or be deflected along the interface. The requirements to achieve the latter failure mode are obtained with the help of conditions based on local stresses or strain energy stored in the composite constituents (He and Hutchinson, 1989; Gupta et al., 1992; Martinez and Gupta, 1994). When the matrix crack tip touches the interface, the singularity exponent is different from the classical value 0.5 and now depends on the elastic properties of the bimaterial constituents (Leguillon and Sanchez-Palencia, 1987). A stiffer matrix or a softer matrix respectively leads to a strong or a weak singularity as the singularity exponent is lower or greater than 0.5. This discontinuous change in the order of the singularity implies that the energy release rate is either infinite or zero in the case of a strong or a weak singularity (Leguillon and Sanchez-Palencia, 1992). Consequently, particular crack extensions must be assumed to assess the competition between deflection and penetration at the interface and the corresponding energy conditions depend on this arbitrary choice (Ahn et al., 1998; Martin et al., 2001).

Nevertheless, several researchers presented physical evidence of tensile debonding ahead of a crack tip (Theocaris and Milios, 1983; Kagawa and Goto, 1998; Barber et al., 2002; Majumdar et al., 1998; Zhang and Lewandowski, 1997). The recent work of Xu et al. (2003) clearly demonstrates that an interfacial crack may be nucleated through the remote interaction of an incoming crack with a weak interface. Cook and Gordon (1964) first analyzed this mechanism by considering an elliptical notch and by postulating that the debonding condition requires that the debond stress at the interface be reached before the cohesive strength is attained at the notch. Kaw and Pagano (1993) solved the stresses and the stress intensity factor in a concentric cylinder with an annular crack under a remote axial strain. The annular crack is located in the vicinity of the interface and a stress condition for interfacial debonding was proposed but the debonding length at initiation was not evaluated. Leguillon et al. (2000) and Martin et al. (2002) investigated the interface debonding ahead of a matrix crack within an asymptotic framework and could derive a condition for debonding in the case of a stiffer matrix. Another analyses postulate the existence of an interfacial defect which grows under the influence of the approaching matrix crack (Heitzer, 1990; Lee et al., 1996; Li, 2000).

The aim of this paper is to provide conditions for the initiation of interfacial failure in the vicinity of a matrix crack in brittle matrix composites. The interface is assumed to be free of defect and a finite fracture mechanics approach is used to describe the nucleation process. It will be shown that the debonding length can be determined with the help of an additional energetic or interfacial strength condition. Considering a stationary matrix crack submitted to a monotonic and increasing loading, the competition between the propagation of the matrix crack and the interfacial debonding can be assessed. The relevant debonding condition compares the ratio of the interfacial toughness over the matrix toughness to a critical value which depends on the elastic properties and the geometry of the bimaterial. If this condition is satisfied, the critical applied deformation at the onset of debonding can be evaluated.

2. Geometry of the cracked bimaterial

The geometry of the cracked bimaterial considered is that of cylindrical cell represented in Fig. 1a. It consists of a single fibre (Young's modulus E_f and Poisson's ratio ν_f) of radius R_f and infinite length surrounded by a concentric cylinder of matrix (Young's modulus E_m and Poisson's ratio ν_m). Poisson 's ratio of the constituents are assumed to be identical with $\nu_f = \nu_m = 0.2$. The inner and outer radii of the matrix are respectively R_f and $\frac{R_f}{\sqrt{V_f}}$ where V_f is the fibre volume fraction. An annular matrix crack is introduced in the plane $z = 0$ and the distance between the crack tip and the fibre/matrix interface is denoted by l. A prescribed displacement is applied on both ends of the so-called microcomposite and the external cylindrical surface of the matrix is stress free.

A linear elastic and brittle behaviour is assumed for each constituent of the bimaterial. The mode of deformation is axisymmetric so that the nonzero stresses σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz}, σ_{rz} and displacement components u_r,

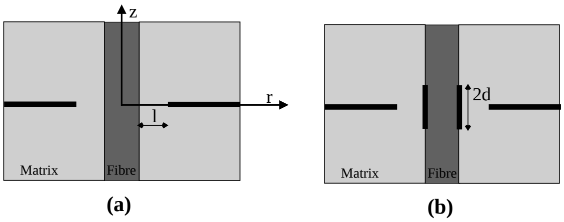

Fig. 1. Geometry of the bimaterial: (a) introduction of an annular crack, (b) nucleation of an interfacial crack in the vicinity of the matrix crack.

u_z only depend on r and z. A finite element model already described elsewhere is used to obtain these components (Martin et al., 1998). The radial stress along the fibre/matrix interface is given by

$$\sigma_{rr}(l,z) = k_{rr}(l,z)\sigma, \tag{1}$$

where σ is the applied stress. Fig. 2 shows that the maximum of the radial stress along the interface increases when the ligament l decreases and that the corresponding profile is modified by the elastic mismatch between the fibre and the matrix. As a result of this stress concentration induced by the matrix crack, an interfacial crack of length $2d$ may nucleate for a given value of the applied stress σ as depicted in Fig. 1b.

The energy release rate for the propagation of this interfacial crack in the vicinity of the matrix crack is denoted by $G_{\mathrm{i}}(l,d)$ with

$$G_{\mathrm{i}}(l,d) = A_{\mathrm{i}}(l,d)\sigma^2. \tag{2}$$

A dedicated numerical procedure (Martin et al., 1998) was developed to estimate the normalised energy release rate $A_{\mathrm{i}}(l,d)$. It is based on a virtual crack closure integral method and uses highly refined meshes in the vicinity of the crack tip. This method was demonstrated to provide accurate results (Buchholz et al., 1999). Closure of the interfacial crack was checked but was not observed for the simulated debond lengths with $\frac{d}{l} \leqslant 10$ and $0.4\% \leqslant \frac{l}{R_{\mathrm{f}}} \leqslant 4\%$. Fig. 3 plots the value of $A_{\mathrm{i}}(l,d)$ versus the decohesion length for three values of the elastic mismatch between the fibre and the matrix. It is noticeable that the normalised energy

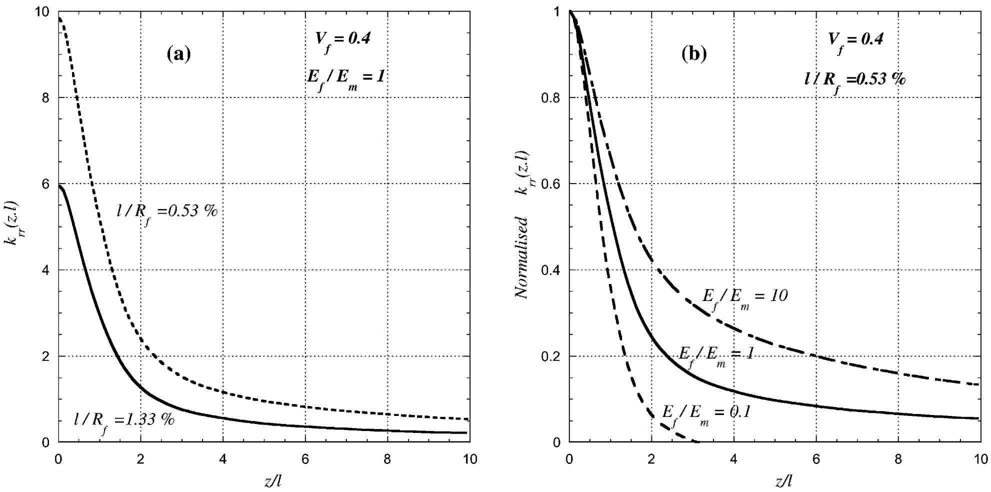

Fig. 2. Radial stress along the interface (a) for different values of the ligament and (b) for different values of the elastic mismatch. In the last case, the values are normalised by the maximum obtained at $z = 0$. The fibre volume fraction is $V_{\mathrm{f}} = 0.4$.

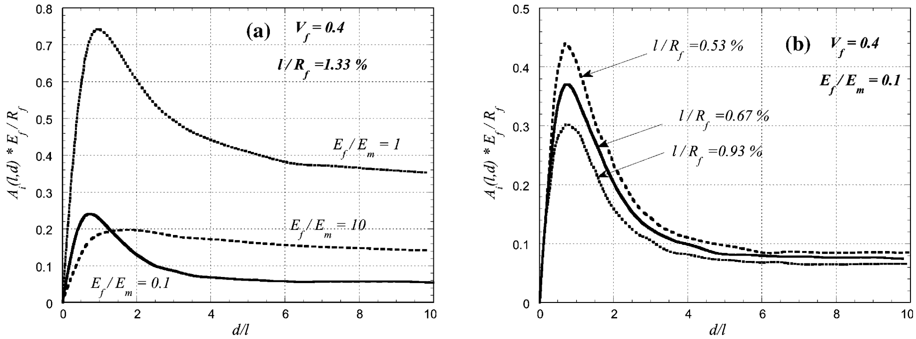

Fig. 3. Adimensional normalised energy release rate $A_i(l,d)$ versus the decohesion length (a) for a given value of the ligament and (b) for a given value of the modulus ratio. The fibre volume fraction is $V_f = 0.4$.

release rate $A_i(l,d)$ always exhibits a peak for $0.8 \leqslant \frac{d}{l} \leqslant 2$ before reaching a nearly steady state value. The presence of this peak indicates that the initiation of debonding in the vicinity of the matrix crack is an unstable phenomenon. The interfacial crack will appear in a very short time leading to a spontaneous crack growth (Leguillon, 1999). As it is not possible to describe in detail this kind of nucleation process, a finite fracture mechanics approach is used (Hashin, 1996).

3. Energetic condition for an interfacial decohesion ahead of a matrix crack

The energetic condition for nucleation of the interfacial crack is derived from the change in potential energy $\Delta W(l,d)$ between the two states schematised in Fig. 1a and b which is given by

$$\Delta W(l,d) = W(l,d=0) - W(l,d), \tag{3}$$

with

$$\frac{1}{2\pi R_f}\frac{\partial W(l,d)}{\partial d} = G_i(l,d). \tag{4}$$

If an interfacial crack of length $2d^*$ is nucleated, an energy balance states that:

$$\Delta W(l,d^*) \geqslant 2\pi R_f d^* G_i^c, \tag{5}$$

where G_i^c is the toughness of the fibre/matrix interface. Combining Eqs. (2) and (4), the change in potential energy can be written as

$$\Delta W(l,d^*) = \int_0^{d^*} 2\pi R_f G_i(l,z)\,dz = \sigma^2 \int_0^{d^*} 2\pi R_f A_i(l,z)\,dz, \tag{6}$$

and the energy balance (5) becomes

$$\sigma^2 \int_0^{d^*} A_i(l,z)\,dz \geqslant G_i^c d^*. \tag{7}$$

Assuming that d^* is different from zero, Eq. (7) can be written as

$$\frac{\sigma^2}{d^*}\int_0^{d^*} A_i(l,z)\,dz \geqslant G_i^c. \tag{8}$$

Introducing

$$\overline{A_{\mathrm{i}}(l,d)} = \frac{1}{d}\int_0^d A_{\mathrm{i}}(l,z)\,\mathrm{d}z, \tag{9}$$

Eq. (8) is equivalent to

$$\sigma^2\overline{A_{\mathrm{i}}(l,d^*)} \geqslant G_{\mathrm{i}}^{\mathrm{c}}, \tag{10}$$

which reveals an incremental condition in which the infinitesimal energy rates of the classical Griffith's condition are replaced by finite energy increments. Use of the energetic condition (10) requires the determination of the decohesion length at nucleation d^*. Furthermore, the stress σ at decohesion is also unknown and an additional condition must be established. As shown in the next sections, this supplementary relation can be derived from the energetic analysis or obtained with the help of the strength condition.

4. Evaluation of the decohesion length with the help of an energetic analysis

The evolution of the potential energy $\Delta W(l,d)$ versus the decohesion length can be deduced from the results presented in Fig. 3. As schematically plotted in Fig. 4, $\Delta W(l,d)$ is concave near $d = 0$ but becomes convex for greater values of d. This curve shape is a consequence of (i) a zero initial slope as $A_{\mathrm{i}}(l,d=0) = 0$, (ii) the presence of an inflexion point corresponding to the maximum of $A_{\mathrm{i}}(l,d)$, (iii) a constant slope for greater values of d. Due to the convexity of $\Delta W(l,d)$ after its inflexion point, the energy balance (5) can be satisfied if the applied stress is increased as illustrated in Fig. 4a.

For this specific value of the applied stress, the equality of the slopes in Fig. 4a implies that the following condition is also satisfied:

$$\left(\frac{\partial W(l,d)}{\partial d}\right)_{d=d^*} = \left(\frac{\partial 2\pi R_{\mathrm{f}} G_{\mathrm{i}}^{\mathrm{c}} d}{\partial d}\right)_{d=d^*}, \tag{11}$$

which turns to

$$\sigma^2 A_{\mathrm{i}}(l,d^*) = G_{\mathrm{i}}^{\mathrm{c}}. \tag{12}$$

Eq. (12) is simply Griffith's condition written for an interfacial crack of size $2d^*$. Combining Eqs. (10) and (12) leads to

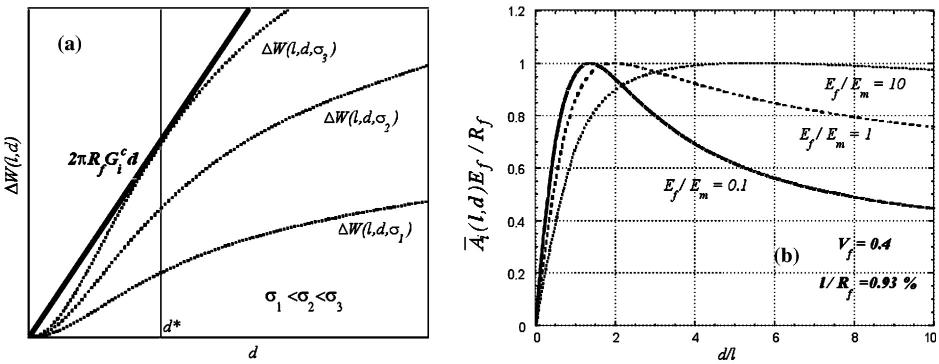

Fig. 4. (a) Schematic evolution of the potential energy change $\Delta W(l,d)$ versus the decohesion length. The applied stress is increased so that the energy balance (10) can be satisfied for $d = d^*$. (b) Adimensional normalised value of $\overline{A_{\mathrm{i}}(l,d)}$ versus the decohesion length for a given value of the ligament. The fibre volume fraction is $V_{\mathrm{f}} = 0.4$.

$$\overline{A_{\rm i}(l,d^*)} = A_{\rm i}(l,d^*). \tag{13}$$

Eq. (13) allows to evaluate the decohesion length d^* and is equivalent to

$$\left(\frac{\partial \overline{A_{\rm i}(l,d)}}{\partial d}\right)_{d=d^*} = 0. \tag{14}$$

It is deduced that the decohesion length d^* is the value $d^* = d^{\rm m}$ which maximises $\overline{A_{\rm i}(l,d)}$ for a given value of the ligament. As indicated in Fig. 4b which plots the normalised value of $\overline{A_{\rm i}(l,d)}$ for a given value of the modulus ratio, computation reveals that $\overline{A_{\rm i}(l,d)}$ always exhibits a maximum for a specific value of the decohesion length.

Eq. (14) was used to evaluate d^* as plotted in Fig. 5 for different values of the ligament. Fig. 5 shows that the debonding length is markedly affected by the elastic mismatch between the fibre and the matrix. A stiffer matrix leads to a smaller debonding zone with $1.5 \leqslant \frac{d^*}{l} \leqslant 2$. In this case, the strong singularity induces a very localised stress concentration on the interface as shown in Fig. 2b. Conversely, the presence of a stiffer fibre increases the debonding zone. This is a consequence of the widening of the interfacial zone affected by the stress concentration for a weak singularity as shown in Fig. 2b.

A similar approach was used by Martin et al. (2002) within an asymptotic framework. In this paper, the energetic condition was identical to relation (10). It was considered that increasing the applied stress will decrease $\frac{G_{\rm i}^{\rm c}}{\sigma^2}$ so that (10) can be fulfilled. The onset of decohesion will thus take place for the maximum of $\overline{A_{\rm i}(l,d)}$ as stated in (14). However, the asymptotic analysis at the leading order could not capture this maximum in the case of a weak singularity ($E_{\rm m} < E_{\rm f}$). The relevant asymptotic values obtained by Martin et al. (2002) are thus only plotted in Fig. 5 for a stiffer matrix. The good correlation between these results and the present analysis shows that the decohesion length is weakly dependent on the loading geometry in the case of a strong singularity. It is interesting to note that Eq. (13) or (14) indicates that the decohesion length $d^*(l, E_{\rm f}, E_{\rm m}, \nu_{\rm f}, \nu_{\rm m})$ depends on the elastic properties and the geometry of the bimaterial but not on the toughness $G_{\rm i}^{\rm c}$. However, the critical stress at decohesion depends on the toughness as shown by Eq. (12). Similar conclusions were also obtained by Bylterist and Marigo (2003) in their efforts to obtain the crack length at initiation for a pull out test. These authors have derived some relations identical to (10) and (12) from a least energy principle.

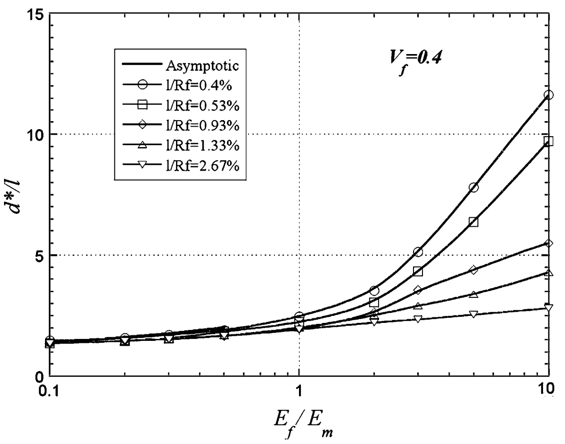

Fig. 5. Decohesion length at nucleation as a function of the modulus ratio as determined with the help of the additional energetic condition (14). The fibre volume fraction is $V_{\rm f} = 0.4$.

5. Evaluation of the decohesion length with the help of a strength condition

As was mentioned in Section 3, the incremental energy condition (10) is only a necessary but not sufficient condition which does not allow to determine the decohesion length d^*. An additional condition is necessary and a strength condition is used in this section. Combined with the incremental energy condition (10), this approach proves efficient to predict crack nucleation in various cases like in monolithic v-notched specimens or in an epoxy joint between two steel plates (Leguillon et al., 2003). As introduced by Leguillon (2002), this condition states that the radial stress along the anticipated path of crack nucleation is greater than the interfacial strength σ_i^c:

$$\sigma_{rr}(l,d^*) = k_{rr}(l,d^*)\sigma \geqslant \sigma_i^c. \tag{15}$$

For a monotonically increasing load, Eq. (10) is first satisfied for $d = d^m$ if

$$\sigma^2 \overline{A_i(l,d^m)} = G_i^c. \tag{16}$$

The decohesion length is thus $d^* = d^m$ if the additional condition (15) is also satisfied which leads to

$$\frac{\overline{A_i(l,d^m)}}{k_{rr}^2(l,d^m)} \leqslant \frac{G_i^c}{(\sigma_i^c)^2}. \tag{17}$$

If Eq. (17) is satisfied, the decohesion length at nucleation has the same value which was determined in the previous section with the help of the additional energetic condition. If Eq. (17) is not satisfied, the applied stress must be increased and the decohesion length $d^* < d^m$ is now given by

$$\frac{\overline{A_i(l,d^*)}}{k_{rr}^2(l,d^*)} = \frac{G_i^c}{(\sigma_i^c)^2}. \tag{18}$$

The second term in (17) or (18) is related to a characteristic size d_i^c of the interface given by

$$d_i^c = \frac{E_i G_i^c}{(\sigma_i^c)^2} \quad \text{with} \quad \frac{2}{E_i} = \frac{1-\nu_f^2}{E_f} + \frac{1-\nu_m^2}{E_m}. \tag{19}$$

E_i is the effective modulus which enters the energy release rate expression for an interfacial crack between two semi-infinite and different elastic materials (Hutchinson et al., 1987). Low toughness and high strength interfaces are associated with low values of the characteristic size while high toughness and low strength interfaces lead to higher values of d_i^c. According to (17), the decohesion length will be identical to the value given in (14) if the characteristic size d_i^c is greater than $E_i \frac{\overline{A_i(l,d^m)}}{k_{rr}^2(l,d^m)}$ which only depends on the elastic properties and geometry of the bimaterial.

Fig. 6 plots the decohesion length d^* as obtained from (17) and (18) for different values of the ligament. As already observed in Fig. 5, the debond length increases with the modulus ratio. As expected, Eq. (17) cannot be satisfied if the value of d_i^c is too low. In this case, the decohesion length is smaller than the value obtained in the previous section as shown in Fig. 6a. This effect is all the more important for a stiffer fibre. In the case of a strong singularity ($E_m > E_f$), the different approaches only provide different values of the decohesion length for very low values of d_i^c as shown in Fig. 6b. It is worthy of note that $d^*(l, d_i^c, E_f, E_m, \nu_f, \nu_m)$ is always a function of the elastic properties and geometry of the bimaterial but now also depends on a characteristic size of the interface.

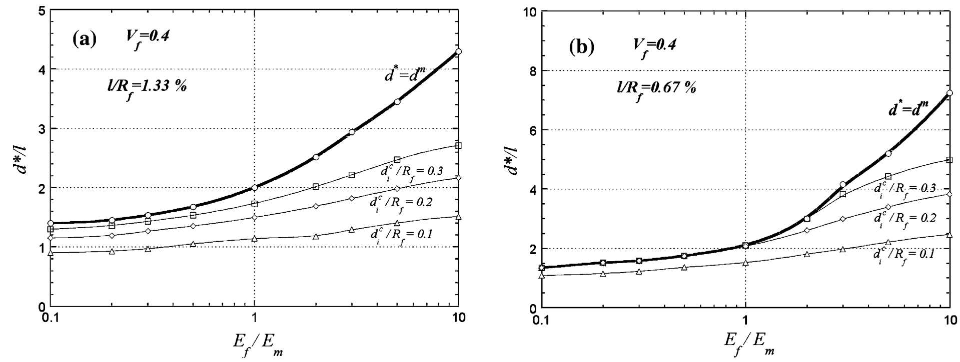

Fig. 6. Decohesion length at nucleation as a function of the modulus ratio as determined with the help of the additional strength condition (17) and (18) for different values of the characteristic size d_i^c. The fibre volume fraction is $V_f = 0.4$.

6. Competition between the matrix crack propagation and the interfacial decohesion

The competition between the matrix crack propagation and the interfacial decohesion is now assessed. The matrix crack is considered as stationary. The applied stress is monotonically increased and the interfacial nucleation will take place preferentially to the propagation of the matrix crack if the following conditions are satisfied:

$$\begin{cases} \sigma^2 A_i(l, d^*) = G_i^c, \\ G_m(l) = A_m(l)\sigma^2 < G_m^c, \end{cases} \tag{20}$$

where $G_m(l)$ is the energy release rate for the propagation of the matrix crack and G_m^c is the matrix toughness. Condition (20) leads to

$$\frac{G_i^c}{G_m^c} < \frac{A_i(l, d^*)}{A_m(l)} \tag{21}$$

which must be fulfilled to promote decohesion.

The normalised energy release rate $A_m(l)$ was evaluated with the help of a virtual crack closure method and Fig. 7a plots the critical ratio $\frac{A_i(l,d^*)}{A_m(l)}$ versus the ratio $\frac{E_f}{E_m}$. In this case, $d^*(l)$ is determined with the help of

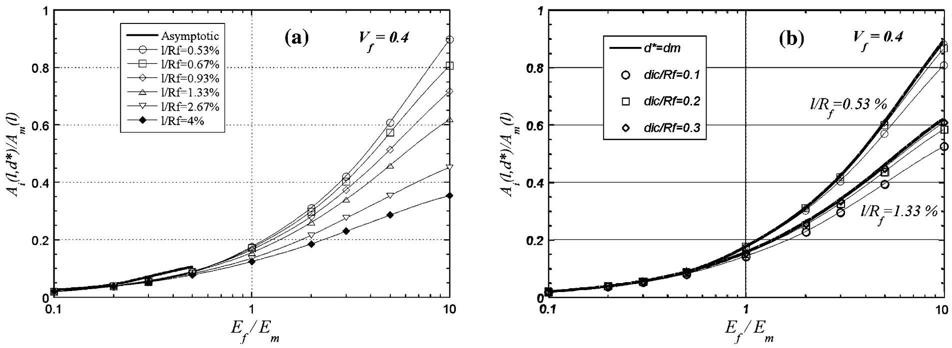

Fig. 7. Critical ratio for nucleation of the debond as a function of the modulus ratio (a) the decohesion length at nucleation is determined with the help of the additional energetic condition, (b) the decohesion length at nucleation is determined with the help of the additional strength condition. The fibre volume fraction is $V_f = 0.4$.

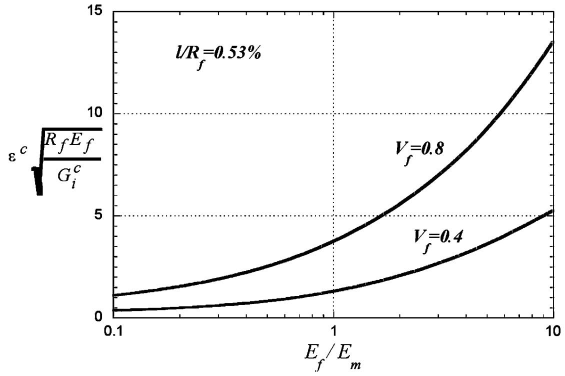

Fig. 8. Critical applied deformation for nucleation of the debond as a function of the modulus ratio. The decohesion length at nucleation is determined with the help of the additional energetic condition.

the additional energetic condition. According to (21), the curves in Fig. 7a delineate domains for interfacial nucleation. For a stiffer matrix, debonding is only possible for a low value of the interfacial toughness whatever the value of the ligament. As logically expected from the tendency observed in Fig. 5, the critical ratio is strongly dependent on the ligament for a stiffer fibre. In terms of toughness, the decohesion is facilitated by a stiffer fibre and a smaller ligament. The asymptotic values obtained by (Martin et al., 2002) are also plotted in Fig. 7a and demonstrate a good agreement with the present values for a stiffer matrix. The results plotted in Fig. 7a are thus weakly dependent on the loading geometry in the case of a strong singularity ($E_m > E_f$).

Fig. 7b gives the relevant results if $d^*(l)$ is now determined with the help of the additional strength condition. In this case, the decohesion length at nucleation also depends on the characteristic size d_i^c as given by Eqs. (17) and (18). Compared with the previous results, it is noticeable that the value of d_i^c has a weak influence on the critical ratio for a stiffer fibre. Compared with the previous results in Fig. 7a, a lower value of the characteristic size does not facilitate the decohesion.

If condition (21) is satisfied, Eq. (20) provides the critical loading for nucleation of the debond. The relevant critical deformation applied on the cylindrical cell is plotted in Fig. 8. The stress concentration induced at the interface by the matrix crack is lowered in the case of a stiffer fibre and leads in this case to a higher critical deformation. Fig. 8 also shows that this effect is amplified by increasing the fibre volume fraction.

7. Discussion

The previous sections only considered the configuration of a stationary matrix crack lying at a given distance l of the interface. The loading is then progressively increased and the competition between the interfacial decohesion and the propagation of the matrix crack is examined. The question now arises to investigate a more realistic situation with a crack propagating in the matrix towards the interface.

In the case of a stiffer fibre (weak singularity), the situation is rather simple because the propagation of the matrix crack is stable (Martin et al., 1998). The applied load must be increased to propagate the crack and the results of the previous sections can be used. As the ligament is reducing due to the propagation of the matrix crack, the interfacial decohesion is facilitated as shown in Fig. 7.

The situation is different in the case of a stiffer matrix (strong singularity) because the propagation of the matrix crack is unstable (Martin et al., 1998). In contrast, it will be shown that the interfacial decohesion is inhibited. If l_0 denotes the initial ligament of the matrix crack, the applied stress at initiation of the matrix crack is given by

$$(\sigma^{\mathrm{c}})^2 = \frac{G_{\mathrm{m}}^{\mathrm{c}}}{A_{\mathrm{m}}(l_0)}, \tag{22}$$

and it will be assumed that the interfacial decohesion is not initially observed so that Eq. (21) is not satisfied with

$$\frac{G_{\mathrm{i}}^{\mathrm{c}}}{G_{\mathrm{m}}^{\mathrm{c}}} > \frac{A_{\mathrm{i}}(l_0, d^*(l_0))}{A_{\mathrm{m}}(l_0)}. \tag{23}$$

After the initiation phase, the crack is unstable and now propagates under the constant applied stress σ^{c} with $l < l_0$ and $A_{\mathrm{m}}(l) > A_{\mathrm{m}}(l_0)$. Ignoring the dynamic effects, a quasi-static analysis is followed. For each value of the ligament l, a decohesion length $d^*(l)$ can be defined with the help of Eq. (13) or (17) and (18). However, Eq. (21) is no more relevant as the crack propagates under a constant applied stress. The competition between the interfacial decohesion and the matrix crack propagation is evaluated by postulating that the crack will follow the path which maximises the energy released by fracture (Martin et al., 2001). The following condition must be satisfied if the interfacial decohesion occurs preferentially to the matrix crack propagation:

$$(\sigma^{\mathrm{c}})^2 \overline{A_{\mathrm{i}}(l, d^*(l))} - G_{\mathrm{i}}^{\mathrm{c}} > (\sigma^{\mathrm{c}})^2 A_{\mathrm{m}}(l) - G_{\mathrm{m}}^{\mathrm{c}}. \tag{24}$$

Combining Eqs. (22) and (24) leads to the condition:

$$1 + \frac{A_{\mathrm{m}}(l)}{A_{\mathrm{m}}(l_0)} \left(\frac{\overline{A_{\mathrm{i}}(l, d^*(l))}}{A_{\mathrm{m}}(l)} - 1 \right) \geqslant \frac{G_{\mathrm{i}}^{\mathrm{c}}}{G_{\mathrm{m}}^{\mathrm{c}}}. \tag{25}$$

It appears that the first term in Eq. (25) is negative because $\frac{A_{\mathrm{m}}(l)}{A_{\mathrm{m}}(l_0)} > 1$ and $\frac{\overline{A_{\mathrm{i}}(l, d^*(l))}}{A_{\mathrm{m}}(l)} < 0.2$ as shown in Fig. 7 for a strong singularity. It is thus concluded that Eq. (25) cannot be satisfied: the interfacial decohesion for a matrix crack propagating towards the interface in the case of a stiffer matrix is unlikely.

8. Conclusion

The onset of interfacial debonding in the vicinity of a matrix crack is analysed in the case of an axi-symmetric fibre/matrix cell submitted to a tensile loading. The interface is assumed to be free of defect and an energetic analysis is used to describe the nucleation process. However, the determination of the decohesion length is essential to establish the deflection condition. Two approaches are then compared: (i) the decohesion length is taken as the value which maximises the potential energy change, (ii) the decohesion length is evaluated with the help of an additional strength condition. In the first case, the decohesion length d^* depends on the geometry of the bimaterial cell while in the second case d^* also depends on the fracture properties of the interface. The two methods lead to similar values of the decohesion length which reveal smaller in the second case for low toughness and high strength interfaces.

A decohesion condition is then derived for a stationary crack submitted to an increasing loading. In the case of a stiffer matrix (strong singularity), the decohesion is only predicted for low toughness interfaces with $\frac{G_{\mathrm{i}}^{\mathrm{c}}}{G_{\mathrm{m}}^{\mathrm{c}}} < 0.02$–$0.2$ depending on the elastic contrast between the fibre and the matrix. The critical ratio is weakly dependent on the ligament value and the results are very close to those provided by a previous asymptotic analysis. In the case of a stiffer fibre (weak singularity), the decohesion is facilitated by decreasing the ligament.

The situation of a crack propagating towards the interface is also investigated. In the case of a weak singularity, the decohesion is always predicted as the crack propagates in a stable way under an increasing load. In the case of a strong singularity, if the decohesion does not occur preferentially to the initiation of the matrix crack, it is shown that the interfacial decohesion is unlikely during the unstable phase of propagation.

Use of the condition for interfacial debonding requires the identification of two material parameters: the interfacial toughness and the interfacial strength. Recent papers have shown that these values can be determined experimentally by comparing failure loads with numerical analysis incorporating cohesive laws (Mohammed and Liechti, 2000; Sorensen, 2002). It is also expected that these parameters can be evaluated with the help of initiation loads obtained from two different micromechanical tests (push-out and pull out experiments for example) performed on a fibre/matrix system.

It is worth noting that a complete analysis should also consider a crack nucleation in the fibre. The competition between the interfacial decohesion and a fibre penetration mechanism is not addressed here but can be analysed with the same approach.

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